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Application of Graph-based model on Nashik district state road network.

Aplicación del modelo basado en grafos en la red de carreteras estatales del distrito de Nashik

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ABSTRACT

The role of graph theory in enabling network analysis and generalization is briefly discussed, and graph representations offer a handy way of handling the topological and related information describing a road network. The importance of obtaining measures of the functional relevance of network road segments given a context described in terms of a set of sites of interest is then demonstrated using graph theoretic techniques such as the shortest path between network nodes and spanning trees. The road network extends from Nashik's central business area to all of the district's tehsils. The goal of this article is to use graph theory to assess the road connectivity and network accessibility of Nashik to all Tehsil headquarters. Furthermore, the Alpha index, Beta index, Gama index, Chromatic number, and Dijkstra's Algorithm have been utilized to categorize network accessibility and determine the least distance between any two tehsils in Nashik District.

Keywords: Graph Theory, Nashik, Road, Network, Analysis, Index, Shortest Path, Algorithm

RESUMEN

Se analiza brevemente el papel de la teoría de grafos para permitir el análisis y la generalización de redes, y las representaciones gráficas ofrecen una forma práctica de manejar la información topológica y relacionada que describe una red de carreteras. La importancia de obtener medidas de la relevancia funcional de los segmentos de carreteras de la red dado un contexto descrito en términos de un conjunto de sitios de interés se demuestra luego utilizando técnicas de teoría de grafos como el camino más corto entre los nodos de la red y los árboles de expansión. La red de carreteras se extiende desde el área comercial central de Nashik hasta todos los tehsils del distrito. El objetivo de este artículo es utilizar la teoría de grafos para evaluar la conectividad vial y la accesibilidad de la red de Nashik a todas las sedes de Tehsil. Además, el índice alfa, el índice beta, el índice gama, el número cromático y el algoritmo de Dijkstra se han utilizado para categorizar la accesibilidad de la red y determinar la distancia mínima entre dos tehsils en el distrito de Nashik.

Palabras clave: teoría de grafos, Nashik, carretera, red, análisis, índice, ruta más corta, algoritmo

INTRODUCTION

Science is an all-encompassing study that investigates the structure and behaviour of the physical and natural world via observation and experimentation (Deboer 2006). Mathematics is a discipline of science that studies numbers, quantities, and spatial relationships (Van Nes and De Lange, 2007). Graph theory is a field in which we use graphs to understand mathematical structure. A graph is made up of vertices (nodes/points) and edges (connections between them) (Lines) (Dale and Fortin 2010). In real life, cities are referred to as vertices, and the distance between them is referred to as an edge (Shi and Kosari 2021). The nation's development depends on the road network.

Traditionally, transportation has been examined as a structural characteristic in regional studies as well as a field in engineering, geography, economics, and planning (Chacon-Hurtado 2020). However, with the advent of the quantitative revolution in Mathematics, a new dimension for transportation study was opened, with a variety of options (Barnes 2008). Graph theory has been used by many social scientists to investigate transportation networks (Zhang 2010).

Lalanne looked at the density and pattern of France's rail and road networks (Kansky and Danscoine 1989). Currie examined a detailed economic survey with a focus on transportation's significance in the Canadian economy (Sharpe and Currie 2008). Garrison examined the highway connection in the United States using graph theory tools (Garrison and Marble 1965). Kissling was interested in learning more about Nova Scotia's roadway system (Cain, 2005). Several scientists used three key indices to undertake network analysis in transportation in the past: the Alpha, Beta, and Gamma indices, which are all indicators of connection (Schnarre 2022). As graph theory developed in the last century, network analysis became a basic concept in transportation research, according to Black (Scheider and de Jong 2022). Rodrigue devised a set of metrics to assess transportation networks, including connectedness, diameter, cyclomatic number, and accessibility (Woldemariam 2019).

Roads are the first, oldest, and most widely used means of transportation. Road transport is India's major mode of transportation in terms of traffic share and contribution to the national economy (GoI MRT&H, 2021). Any country's socio-economic development is dependent on its road network. The entire length of the world's road network has reached 64 million kilometers, and the number of vehicles on the road has increased to 1.78 billion (UN ESCAP, 2016). With a length of 62, 15,797 kilometers as of March 31, 2018, India's road network is the second largest in the world. Maharashtra is the top-ranking state, with a total road network length of 626491 km (11.79 percent), followed by Uttar Pradesh (8.21 percent), and Madhya Pradesh (8.21 percent) (6.85 percent). In the current attempt, the principal road network of tehsil nodes has been assessed.

The road network in India has the second-largest road network in the world. The total length on 31st March 2019 is 63,71,847 KM. The breakup of major categories is shown in figure 1.

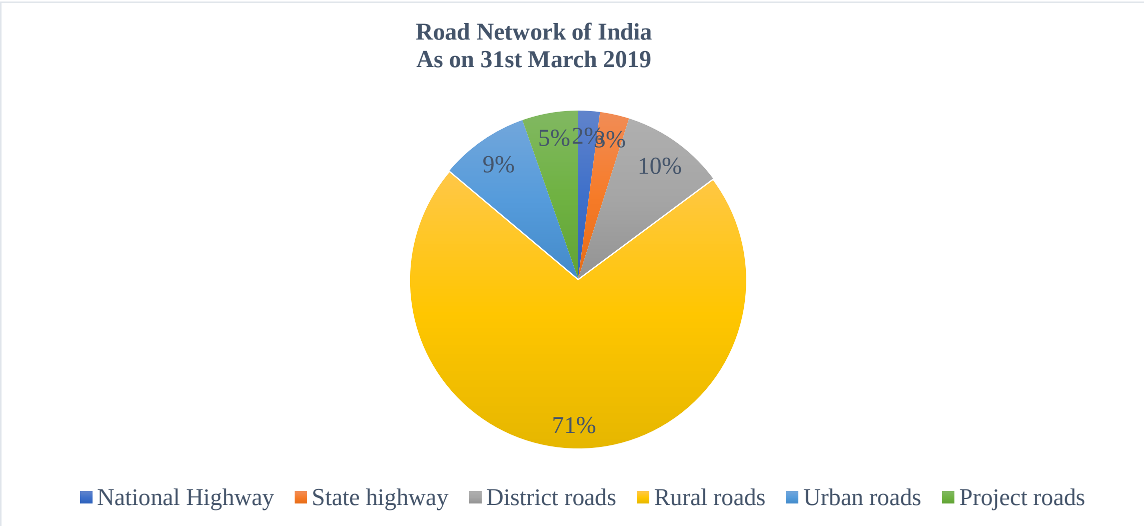

Figure 1: Road network Distribution in India

Table 1: Road network length of Maharashtra and Nashik district.

Road Categories	Road Network (Length In KM)			
	Maharashtra state	%	Nashik district	%
National Highway	4509.09	1	254.08	3
State highway	35754.68	11	686.52	10
Major District roads	51993.71	16	1054.06	15
Other district roads	61158.56	19	1210.18	17
Rural roads	175731.48	53	3916.37	55
Total	336182.56	100	7121.21	100

Source: (cultural.maharashtra.gov.in, surface classification in Nashik district in 1971-72)

The Indian state of Maharashtra's northern region contains Nashik. The fourth-largest city in Maharashtra is Nashik. Nashik is well renowned for being one of the 12-year Kumbh Mela pilgrimage centres for Hindus. Nashik is located in the northern region of Maharashtra State at a height of 584 m (1,916 ft) above mean sea level, which provides it with optimal seasonal temperature variance. National Highway 60 is used to get to Nashik (Chakrabarty 2009). The overall length of various road networks in Maharashtra is shown in table1.

The origins of Graph Theory can be traced all the way back to 1735. L. Euler, a Swiss mathematician, is known as the "Father of Graph Theory". The Swiss mathematician L. Euler is credited with being the father of graph theory. It was widely applied to the road network in the 1960s and 1970s. Edward mentioned that graph theory is a branch of topology dealing with abstract configurations of points and lines or nodes and links (Heckmann 2015). The transport geographer William introduced the elementary and descriptive use of graph theory to assess the effects of changes in the transport network. Kansky Studied transport networks with applications of graph theory and established ratio and non-ration indices, i.e. alpha (α), beta (β), gamma (γ) & cyclomatic number (μ), network diameters (d) (Wang Wei and Jin 2020). He introduced related count, degree of connectivity, scatter, accessibility, and degree of switching as indicators to measure accessibility. Haggett & Chorley studied network structure and graph theory. Therefore, traffic network analysis to understand structural properties with the help of graph theory is a central topic of traffic geography (Chorley and Haggett 2013).

The degree of completeness of connections between nodes is referred to as network connectivity (Namtirtha and Dutta 2020). While Taaffe & Gauthier stated that the degree of connection between all nodes is a connectivity of networks. Connectivity is a means of measuring the Efficiency of networks and quantitative comparison of different networks. The greater the degree of connectivity within a transportation network, the more efficient the network (Zhang 2015). In this experiment, the connectivity of the state highway network with respect to Tehsils was measured using graph theory.

As preliminaries a graph captures the basic topological structure of a network: a collection of vertices or nodes and a set of edges or arcs, each of which connects two vertices. The number of edges incident at a vertex determines its valency (Raza 2021). A path is a connection of edges while moving from the start to the end vertex. A graph is linked if there is a path connecting any two of its vertices. A cycle is a path that leads from one vertex to another where the start and end vertex is the same. A tree is a linked graph that does not have any cycles. A spanning tree is a subgraph that includes all vertices of a graph with the least number of edges (Bogdanowicz 2020). Here, graphs can depict the topology of road networks in a natural fashion, with vertices corresponding to road junctions, intersections, dead-ends, and areas of interest on the roads such as villages/cities, and edges corresponding to road lengths between such points. To indicate road length or travel time, associating cost values with each edge can improve the representation.

MATERIAL AND METHODS

The description of the arrangement of nodes, their interactions, and the lines or links that connect them makes network analysis a crucial component of transport geography. It provides accessibility and connectivity metrics and enables comparisons across regional networks both inside and between nations. Between a road network and a graph structure, there is a natural relationship. In the absence of further data, the road segments

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most likely to be used in traveling between a set of special interest nodes in a road network, with associated arc costs giving distance or journey time information, can be reasonably believed to be those that make up between the nodes to find the accessibility (Ticha 2017).

The fact that spanning trees have been established for the node set of interest can also be exploited as a means of maintaining connectivity amongst the nodes during generalization and density reduction of the network: by ensuring the retention of the edges of a spanning tree during generalization no vertex becomes isolated from the rest of the network, and so the map conveys the notion of connectivity between features. For this, the minimum cost-spanning tree (MCST) has been proposed.

The data presented in table 1, the indices mentioned were used to measure the connectivity.

Figure 2: road networking between tehsils of Nashik

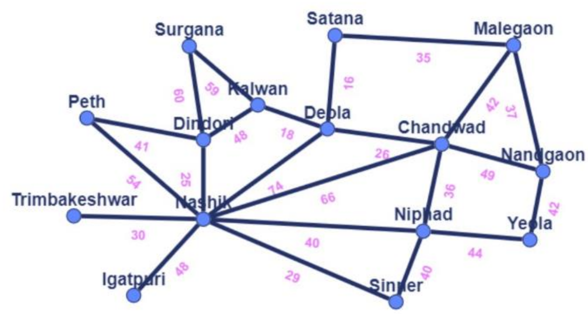

Figure 3: road networking graph of Nashik

1. Beta Index (β)

It is the ratio of edges (e) and nodes (v) of the network.

The equation is β = edges/ nodes = 23/15 = 1.53.

The Impact Index range is from 0 to 1 and >1, the higher the index value, the better the connectivity and the more efficient. The derived index value of 1.53 is greater than one, therefore the connectivity of the network is high, but does not preclude the actual perception of connectivity, which is why another index needs to be calculated.

2. Cyclomatic number (μ)

The actual-non-ratio index measures network connectivity. This is the explicit number of circuits in the network. The greater the number of edges in the network, the greater the number of circuits. The formula is μ = e - v + p; where p is the no. of the unconnected subgraphs. In the network, there is only one connected component (p = 1), so index = 23 - 15 + 1 = 13. This indicates good connectivity of the network.

3. Alpha Index (α)

This is important to compare network connectivity. It is a ratio of actual circuits and the maximum possible circuits in the network. The higher the index, the more network is connected. A tree or simple network always has an index value of 0. The index equation relates to possible circuits in the network and is expressed as:

$$\alpha = (e - v + 1) / (2v - 5)$$

Putting numbers derived from and v index is $\alpha = 15 / (2 * 15 - 5) = 15 / 25 = 0.6$ then when multiplied by 100, the expression would be in percent (%), i.e., 60%. The index shows 60% network connectivity in relation to possible circuits in the given network. This means more circuits are possible to complete the network and it improves the connectivity of the network. If the index value is 1 (100%), it means the network is fully connected.

4. Gamma Index (γ)

This index is the ratio of the number of observed links (edges) and the number of possible links in the network. It is expressed as $\gamma = e / [3 (v-2)]$.

The calculation of the index for the current network is $\gamma = 23/ [3(15-2)] = 0.5897$

If the gamma index value is multiplied by 100, it would be in percent (%). Accordingly, the index is 58.97%, showing that the network connectivity in terms of possible edges in the network is 58.97%. The range of the index is from 0 (0%) poor connectivity to 1 (100%) fully connected.

5. Dijkstra's Algorithm

In a graph, there can be multiple paths that can lead from one source to a destination. These paths may cover different nodes and edges along the path (Javaid 2013). Here, we wish to connect any two tehsils in the Nashik district through a route such that the distance travelled is minimal. This can be achieved through Dijkstra's algorithm. It helps to calculate the shortest path from a single source to all other vertices in the graph.

Algorithm

Step 1: Create two arrays named visited and minDist which check if a node is visited and keep track of the minimum distance to the node from the source respectively.

Step 2: Initially all the nodes are unvisited except the source node. Thus, the visited array will have a false value for all the nodes except the root node.

Step 3: The minimum distance to the source node is 0. Hence, it needs to be marked 0 in the minDist array and the distance to all other nodes should be marked as infinity.

Step 4: Pick the node that is unvisited and has the minimum distance.

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Step 5: Mark the picked node as visited and calculate the minDist of its neighbouring nodes which are unvisited.

Step 6: minDist(neighbour) = minDist(previous node) + weight(connecting edge)

Step 7: Repeat steps 4 to 6 until we reach the destination.

Example 1: Find the shortest path from Peth to Nandgaon

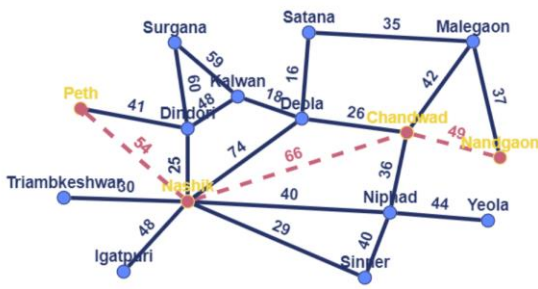

Figure 4: Shortest path from Peth to Nandgaon

Here, we have used Dijkstra's Algorithm to find the shortest path between Peth and Nandgaon. The path is as follows: Peth ⇒ Nashik ⇒ Chandwad ⇒ Nandgaon

The path length is: 169 km

Example 2: Find the shortest path from Igatpuri to Malegaon

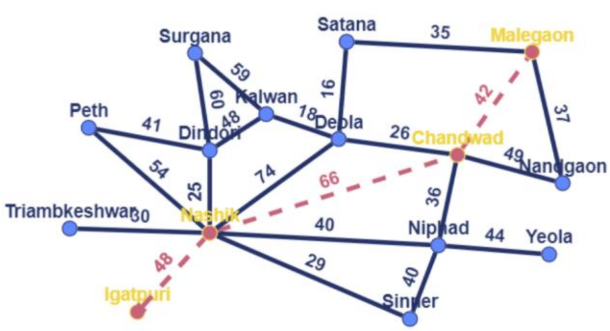

Figure 5: Shortest path from Igatpuri to Malegaon

The path is as follows: Igatpuri ⇒ Nashik ⇒ Chandwad ⇒ Malegaon

The path length is: 156 km

Example 3: Find the shortest path from Niphad to Surgana

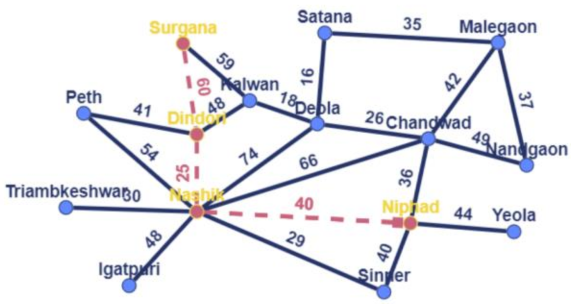

Figure 6: Shortest path from Igatpuri to Malegaon

The path is as follows: Niphad ⇒ Nashik ⇒ Dindori ⇒ Surgana

The path length is: 125 km

RESULTS AND DISCUSSION

Graph theory indices were applied and index values were derived to understand the degree of connectivity of the Nashik district state road network. A total of three ratios (α, β & γ) and one non-ratio index (μ) were applied to the state road network and the index value was generated to derive the degree of connectivity. The expected computation was performed to produce graph theory ratios index values such as beta index (1.53), alpha index (0.6), gamma index (0.5897), and cyclomatic numbers with non-ratio index (13). These indices provide information about the degree of connectivity. It has been observed that the state road network between Nashik district headquarters is not a complete network. The Beta index value is 1.53, which is more than one, but the road network is not complete. In order to improve the understanding of road network connectivity, further indices were calculated. The index alpha (α) is 0.6 (60%) and the gamma (γ) index is 0.5897 (58%). It shows that the connectivity of the state road network of the countries between the targets is not complete 60% in terms of possible circuits and 58% in terms of possible edges in the network (Saxena, 2005). The cyclomatic number describes circuits in the network and the default network index is only 13 circuits.

Table 2 gives the shortest distances calculated from Nashik to all other tehsils present in the Nashik district through Dijkstra's Algorithm. Here, we can observe that most of the paths have common prefixes which means Dijkstra's algorithm saves a lot of computation time by computing the shortest distance to all tehsils at once.

Table 2. Shortest paths from Nashik to all other tehsils

Source	Destination	Shortest Path	Distance
Nashik	Peth	Nashik ⇒ Peth	54
Nashik	Dindori	Nashik ⇒ Dindori	25
Nashik	Triambkeshwar	Nashik ⇒ Triambkeshwar	30
Nashik	Igatpuri	Nashik ⇒ Igatpuri	48
Nashik	Surgana	Nashik ⇒ Dindori ⇒ Surgana	85
Nashik	Kalwan	Nashik ⇒ Dindori ⇒ Kalwan	73
Nashik	Deola	Nashik ⇒ Deola	74
Nashik	Satana	Nashik ⇒ Deola ⇒ Satana	90
Nashik	Chandwad	Nashik ⇒ Chandwad	66
Nashik	Niphad	Nashik => Niphad	40
Nashik	Sinner	Nashik ⇒ Sinner	29
Nashik	Yeola	Nashik ⇒ Niphad ⇒ Yeola	84
Nashik	Nandgaon	Nashik ⇒ Chandwad ⇒ Nandgaon	115
Nashik	Malegaon	Nashik ⇒ Chandwad ⇒ Malegaon	108

As conclusion, the ratio and non-ratio indexes of graph theory were evaluated. The state road network between the HQs of Tehsil in Nashik district is not a complete network. The degree of connectivity is lower as the alpha (α) and gamma (γ) indices show the possibilities for more circuits and edges in the district network. True, the beta index is more cyclomatic number 13, but even more, circuits are possible in the network. This study shows that there is scope for more state road links to complete the region's development network. Dijkstra’s Algorithm has been used on the graph to find the shortest between any two Tehsils. Here, we have demonstrated the use of Dijkstra’s Algorithm to get the shortest paths from Nashik tehsil to all other tehsils in the district.

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